

ARITHMETIC PROGRESSION (A.P.)

Complete Concept Notes • Tinku Sir Tricks • OnePath Hacks • 2023–2026 Board PYQs

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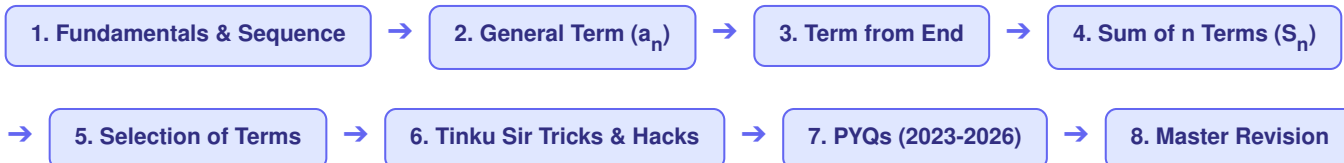
1. CHAPTER OVERVIEW & IMPORTANCE

CHAPTER OVERVIEW What is Arithmetic Progression?

An Arithmetic Progression (A.P.) is a sequence of numbers in which the difference between any two consecutive terms is always constant. This constant value is known as the **common difference** (d).

PARAMETER	DETAILS & CBSE EXAM INSIGHTS
CBSE Board Weightage	4 to 7 Marks (Directly asked in Section B, C, D, or Case Study Section E)
Difficulty Level	Moderate & High Scoring (Formula-driven; requires care in signs & word problems)
Real-World Applications	Financial EMI payments, ladder steps, taxi fare calculations, seating arrangements, salary increments.
Key High-Yield Areas	- Finding n^{th} term from end. - Word problems involving S_n. - Finding a_n from S_n using $a_n = S_n - S_{n-1}$. - Selection of 3, 4, or 5 terms in A.P.

2. CHAPTER ROADMAP



3. COMPLETE CONCEPT COVERAGE

CONCEPT 1: Definition & Identification of an A.P.

 **What is it?** A list of numbers where each term is obtained by adding a fixed number d to the preceding term, except the first term a .

 **DEFINITION** A sequence $a_1, a_2, a_3, \dots, a_n$ is called an **Arithmetic Progression (A.P.)** if and only if:

$$a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1} = d \text{ (Common Difference)}$$

Visualizing Common Difference on a Number Line / Step Chart:



★ KEY PROPERTIES OF 'D'

d can be **Positive** (Increasing A.P. e.g., 2, 5, 8...)

d can be **Negative** (Decreasing A.P. e.g., 10, 7, 4...)

d can be **Zero** (Constant A.P. e.g., 5, 5, 5...)

CONCEPT 2: General Term (n^{th} Term) of an A.P.

 **General Term Formula:** $a_n = a + (n - 1)d$

Where:

$a_n = n^{\text{th}}$ term (or last term l)

a = First term

n = Number of terms ($n \in \mathbb{N}$, must always be a positive integer!)

d = Common difference

TINKU SIR TRICK **Instant Term Calculator:**

Notice the coefficient of d is always 1 less than the term index!

• $a_{10} = a + 9d$ • $a_{25} = a + 24d$ • $a_{100} = a + 99d$

Difference Trick: $a_m - a_k = (m - k)d$. (Extremely useful for solving simultaneous linear equations fast!)

 **COMMON MISTAKE** Never write n as a fraction or negative number. If a question asks "Is 184 a term of 3, 7, 11...", solve for n . If n is not a whole positive integer, 184 is **NOT** a term!

CONCEPT 3: n^{th} Term from the End

 **n^{th} Term from the End:** $a_{n(\text{end})} = l - (n - 1)d$

Where l is the last term of the finite A.P. and d is the common difference of the original forward sequence.

ONEPATH HACK **Reverse the A.P. Method:**

Instead of memorizing a new formula, simply reverse the entire A.P.!

Original A.P.: $a, a+d, \dots, l-d, l$

Reversed A.P.: $l, l-d, \dots, a+d, a$

Now, treat l as your new first term ($A = l$) and $-d$ as your new common difference ($D = -d$). Then use standard $a_n = A + (n-1)D$.
Zero confusion!

CONCEPT 4: Sum of First n Terms (S_n)

The sum of the first n terms of an A.P. is given by two primary formulas:

Form 1: When a , d , and n are given

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Form 2: When First term (a) and Last term (l) are given

$$S_n = \frac{n}{2} [a + l]$$

🔥 CRITICAL RELATION BETWEEN S_n AND a_n

$$a_n = S_n - S_{n-1} \quad (\text{for } n \geq 2) \quad \text{and} \quad a_1 = S_1$$

CBSE Secret: If S_n is given as a quadratic polynomial in n (e.g., $S_n = An^2 + Bn$), then the common difference $d = 2A$ (twice the coefficient of n^2)!

CONCEPT 5: Selection of Terms in A.P. (For Problem Solving)

When the sum of terms is given in a word problem, select terms symmetrically to cancel out d easily:

NO. OF TERMS	SYMMETRICAL SELECTION OF TERMS	COMMON DIFF.
3 Terms	$(a - d), a, (a + d)$	d
4 Terms	$(a - 3d), (a - d), (a + d), (a + 3d)$	$2d$
5 Terms	$(a - 2d), (a - d), a, (a + d), (a + 2d)$	d

4. 📌 COMPLETE FORMULA SHEET

S.N.	CONCEPT / FORMULA	FORMULA EXPRESSION	KEY VARIABLES & CONDITIONS
1	Common Difference	$d = a_k - a_{k-1}$	Must be constant for all adjacent terms.
2	n^{th} Term from Beginning	$a_n = a + (n-1)d$	a = 1st term, d = diff, $n \in \mathbb{Z}^+$
3	n^{th} Term from End	$a_{n(\text{end})} = l - (n-1)d$	l = last term of A.P.
4	Sum of First n Terms (Standard)	$S_n = \frac{n}{2} [2a + (n-1)d]$	Use when a , d , n are available.
5	Sum of First n Terms (Last Term)	$S_n = \frac{n}{2} [a + l]$	Use when 1st term and last term are known.
6	n^{th} Term from Sum	$a_n = S_n - S_{n-1}$	Valid for $n \geq 2$; Note $a_1 = S_1$.
7	Sum of First n Natural Numbers	$S_n = \frac{n(n+1)}{2}$	For sequence 1, 2, 3, ..., n .
8	Sum of First n Even Numbers	$S_n = n(n+1)$	For sequence 2, 4, 6, ..., $2n$.

9	Sum of First n Odd Numbers	$S_n = n^2$	For sequence 1, 3, 5, ..., $(2n - 1)$.
10	Arithmetic Mean (A.M.)	$b = (a + c)/2$	If a, b, c are in A.P.

5. ⚡ TINKU SIR TRICKS

⚡ TRICK #1: Direct Common Difference from S_n

Problem Type: Given $S_n = An^2 + Bn$, find common difference d .

Normal Method: Calculate S_1 , calculate S_2 , find $a_1 = S_1$, $a_2 = S_2 - S_1$, then $d = a_2 - a_1$. (Takes 2 minutes).

Tinku Sir Trick: $d = 2 \times (\text{Coefficient of } n^2) = 2A$

Example: If $S_n = 3n^2 + 5n \rightarrow$ Coefficient of n^2 is 3. So, $d = 2 \times 3 = 6$! (Done in 2 seconds in MCQs!).

⚡ TRICK #2: Middle Term of a Finite A.P.

Problem Type: Find the middle term of an A.P. with N terms.

Tinku Sir Trick:

- If N is **ODD** $\rightarrow$ Single middle term at position $(\frac{N+1}{2})$.
- If N is **EVEN** $\rightarrow$ Two middle terms at positions $(\frac{N}{2})$ and $(\frac{N}{2} + 1)$.

⚡ TRICK #3: Ratio of Sums to Ratio of Terms

Problem Type: Given ratio of sum of n terms of two A.P.s: $\frac{S_{n1}}{S_{n2}} = f(n)$. Find ratio of their m^{th} terms $\frac{a_{m1}}{a_{m2}}$.

Tinku Sir Trick: Replace n by $(2m - 1)$ in the given sum ratio formula!

6. 🚀 ONEPATH HACKS

HACK #1 The Verification Hack for S_n word problems:

Always verify your final n by calculating S_1 . S_1 MUST equal the first term a . If $S_1 \neq a$, your formula or calculation is wrong!

HACK #2 Handling Double Answers in S_n :

When solving $S_n = k$, you often get a quadratic in n yielding two positive integer values (e.g., $n = 5$ and $n = 12$).

Why does this happen? The common difference d is negative! The sum increases, reaches a peak, and then the additional negative terms cancel out the positive terms, making the sum equal again. Always explain this in CBSE exams to score full reasoning marks!

HACK #3 Avoiding Sign Traps in Negative d :

When d is negative (e.g., $d = -3$), always put it in brackets: $a + (n-1)(-3)$. Never write $a + n - 1 - 3$! Over 40% of students lose marks due to missing brackets.

7. GRADED SOLVED EXAMPLES

EXAMPLE 1: Check whether -150 is a term of the A.P. 11, 8, 5, 2...

LEVEL 1: BASIC

Given: A.P. = 11, 8, 5, 2... First term $a = 11$, Common diff $d = 8 - 11 = -3$.

To Find: Is $a_n = -150$ possible for an integer n ?

Step-by-step Solution:

Using general term formula: $a_n = a + (n - 1)d$

$$\rightarrow -150 = 11 + (n - 1)(-3)$$

$$\rightarrow -150 - 11 = (n - 1)(-3)$$

$$\rightarrow -161 = -3(n - 1) \rightarrow n - 1 = \frac{161}{3} = 53.66$$

$$\rightarrow n = 54.66 \text{ (Fractional value)}$$

Final Conclusion: Since n is not a positive integer ($n \notin \mathbb{N}$), **-150 is NOT a term of this A.P.**

EXAMPLE 2: Find the 11th term from the last term of the A.P.: 10, 7, 4, ..., -62.

LEVEL 2: NCERT

Given: A.P. = 10, 7, 4, ..., -62. First term $a = 10$, $d = 7 - 10 = -3$, Last term $l = -62$.

Step-by-step Solution (OnePath Hack Method):

Reverse the A.P.: -62, -59, ..., 4, 7, 10.

New first term $A = -62$.

New common difference $D = +3$ (opposite of original $d = -3$).

We need 11th term of this new sequence:

$$a_{11} = A + (11 - 1)D = -62 + 10(3) = -62 + 30 = -32.$$

Final Answer: The 11th term from the end is **-32**.

EXAMPLE 3: How many terms of the A.P. 24, 21, 18... must be taken so that their sum is 78?

LEVEL 3: CBSE EXAM

Given: $a = 24$, $d = 21 - 24 = -3$, $S_n = 78$.

Step-by-step Solution:

$$\text{Formula: } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\rightarrow 78 = \frac{n}{2} [2(24) + (n - 1)(-3)]$$

$$\rightarrow 156 = n [48 - 3n + 3] = n [51 - 3n]$$

$$\rightarrow 156 = 51n - 3n^2 \rightarrow \text{Divide by 3: } n^2 - 17n + 52 = 0$$

Factoring quadratic equation: $(n - 4)(n - 13) = 0 \rightarrow n = 4 \text{ or } n = 13$.

Explanation for Double Answer: Both values are valid because the sum of terms from 5th to 13th term is 0 (since d is negative and terms become negative).

EXAMPLE 4: Case-Study - Construction of Ladder Steps

LEVEL 4: COMPETENCY BASED

Scenario: A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. The distance between top and bottom rungs is 2.5 m.

Step 1: Find total number of rungs (n):

Total height = 2.5 m = 250 cm. Gap between rungs = 25 cm.

$$n = \left(\frac{250}{25}\right) + 1 = 10 + 1 = 11 \text{ rungs.}$$

Step 2: Find total length of wood required (S_{11}):

First term $a = 45$ cm, Last term $l = 25$ cm, $n = 11$.

$$S_{11} = \frac{11}{2} [a + l] = \frac{11}{2} [45 + 25] = \frac{11}{2} \times 70 = 11 \times 35 = 385 \text{ cm.}$$

Final Answer: Total wood needed = **3.85 meters (or 385 cm)**.

8. QUESTION TYPE MASTER GUIDE

IF QUESTION ASKS...	IDENTIFIED CONCEPT	BEST FORMULA / APPROACH
"Which term of A.P. is X?"	Finding term index n	Set $a_n = X$, solve $n = \frac{(X - a)}{d} + 1$
"Find term from end"	Reverse A.P.	Reverse sequence, use $a_n = l + (n - 1)(-d)$
"Sum of n terms given as polynomial"	Deriving A.P. from S_n	$a_n = S_n - S_{n-1}$ or use Trick $d = 2A$
"3 numbers in A.P. whose sum is..."	Term Selection	Assume terms as $(a - d)$, a , $(a + d)$
"How many terms needed for sum X?"	Quadratic in n	Form $S_n = X \rightarrow$ Solve quadratic $An^2 + Bn + C = 0$

9. CBSE PYQ MASTER SECTION (2023 - 2026)

CBSE 2023 [3 Marks - Official Board Exam]

Question: The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.

Solution:

According to given condition: $a_{17} - a_{10} = 7$

Using $a_n = a + (n - 1)d$:

$$[a + 16d] - [a + 9d] = 7$$

$$\rightarrow a + 16d - a - 9d = 7$$

$$\rightarrow 7d = 7 \rightarrow d = 1.$$

Final Answer: Common difference $d = 1$.

CBSE 2024 [4 Marks - Case Study / Standard Paper]

Question: If the sum of first n terms of an A.P. is given by $S_n = 2n^2 + 3n$, find its 15th term.

Solution:

Method 1 (Standard):

$$a_{15} = S_{15} - S_{14}$$

$$S_{15} = 2(15)^2 + 3(15) = 2(225) + 45 = 450 + 45 = 495$$

$$S_{14} = 2(14)^2 + 3(14) = 2(196) + 42 = 392 + 42 = 434$$

$$a_{15} = 495 - 434 = 61.$$

Tinku Sir Trick Verification: $d = 2(2) = 4$. $a_1 = S_1 = 2(1)^2 + 3(1) = 5$.

$$a_{15} = a_1 + 14d = 5 + 14(4) = 5 + 56 = 61. \text{ (Matches instantly!)}$$

CBSE 2025 [3 Marks - Sample / Board Pattern]

Question: Find the sum of all 3-digit natural numbers which are divisible by 7.

Solution:

• Smallest 3-digit number divisible by 7 = 105.

• Largest 3-digit number divisible by 7 = 994.

Sequence: 105, 112, 119, ..., 994.

Here $a = 105$, $d = 7$, $a_n = 994$.

$$\text{Find } n: 994 = 105 + (n - 1)7 \rightarrow 889 = 7(n - 1) \rightarrow n - 1 = 127 \rightarrow n = 128.$$

$$\text{Find sum } S_{128}: S_{128} = \frac{128}{2} [105 + 994] = 64 \times 1099 = 70,336.$$

Final Answer: 70,336.

PYQ-STYLE QUESTION (CBSE 2026 PREDICTION) [5 Marks - High Probability]

Question: The ratio of the 11th term to the 18th term of an A.P. is 2:3. Find the ratio of the 5th term to the 21st term, and the ratio of sum of first 5 terms to first 21 terms.

Solution:

$$\text{Given } a_{11}/a_{18} = 2/3 \rightarrow (a + 10d)/(a + 17d) = 2/3$$

$$\rightarrow 3(a + 10d) = 2(a + 17d) \rightarrow 3a + 30d = 2a + 34d \rightarrow a = 4d.$$

(i) Ratio of 5th to 21st term:

$$a_5/a_{21} = (a + 4d)/(a + 20d) = (4d + 4d)/(4d + 20d) = 8d/24d = 1/3 \rightarrow 1 : 3.$$

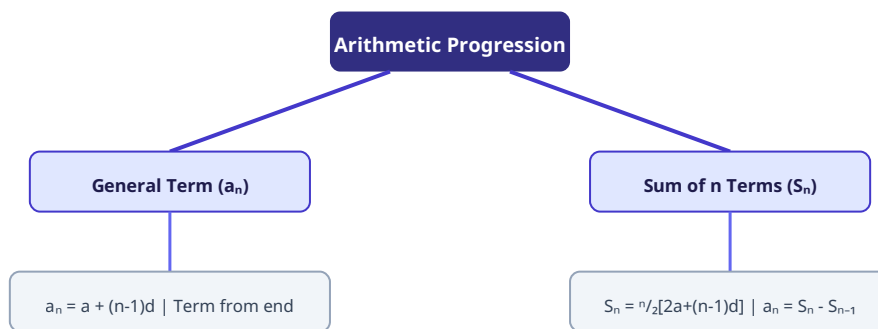
(ii) Ratio of S_5 to S_{21} :

$$S_5/S_{21} = [5/2(2a + 4d)] / [21/2(2a + 20d)] = 5(8d + 4d) / 21(8d + 20d) = 5(12d) / 21(28d) = 60 / 588 = 5 / 49 \rightarrow 5 : 49.$$

10. TOP 10 COMMON MISTAKES STUDENTS MAKE

#	COMMON MISTAKE	WHY IT HAPPENS	CORRECT APPROACH
1	Writing $d = a_1 - a_2$ instead of $a_2 - a_1$	Carelessness in sign order	Always subtract previous term from next term ($a_2 - a_1$).
2	Assuming n can be negative/fraction	Forgetting definition of term position	n MUST be a natural number (1, 2, 3...). Reject invalid roots.
3	Forgetting brackets for negative d	Typographical error in execution	Write $a + (n - 1)(-d)$ to avoid multiplication/subtraction confusion.
4	Confusing a_n (term value) with n (term position)	Concept confusion	n is the position (e.g., 10th seat), a_n is person sitting on it.
5	Calculating $a_n = S_n - S_{n+1}$	Formula misremembering	Correct rule: $a_n = S_n - S_{n-1}$.
6	Miscounting terms in divisibility problems	Boundary condition error	Always use $a_n = a + (n - 1)d$ to solve for exact n .
7	Taking 3 terms in A.P. as $a, a+d, a+2d$ in sum problems	Inefficient variable choice	Use $(a-d), a, (a+d)$ so d cancels out immediately upon adding.
8	Forgetting +1 when counting total runs/steps	Interval miscount	Number of items = $(\text{Total Length} / \text{Gap}) + 1$.
9	Ignoring double answers in quadratic S_n	Assuming one root is wrong	Check if terms become negative. Both positive integer roots can be valid!
10	Leaving answer without units in word problems	Exam rush	Always mention units (cm, rupees, meters, days) in final answer!

11. CONCEPT CONNECTION MAP



12. CBSE EXAM STRATEGY — ARITHMETIC PROGRESSION

PRESENTATION & TIME MANAGEMENT

- **Step 1: Always declare variables first!** Write: "Let 'a' be the first term and 'd' be the common difference of the A.P." (Yields 1/2 mark in board marking scheme).
- **Step 2: State the formula explicitly** before substituting numbers. Board examiners allocate specific steps for formulas.
- **Step 3: Highlight the final answer** inside a clean box with proper units (e.g., **Ans: d = 5**).
- **Step 4: Case-Study Strategy:** Read the contextual paragraph quickly, identify **a** and **d** immediately from the data, then apply formulas. Do not get overwhelmed by real-world story text!

13. ONE-PAGE CHAPTER REVISION SHEET

ARITHMETIC PROGRESSION — QUICK REVISION SNAPSHOT

CORE FORMULAS:

- $a_n = a + (n-1)d$
- $a_{n(\text{end})} = l - (n-1)d$
- $S_n = \frac{n}{2} [2a + (n-1)d]$
- $S_n = \frac{n}{2} [a + l]$
- $a_n = S_n - S_{n-1}$

TINKU SIR TRICKS:

- If $S_n = An^2 + Bn \rightarrow d = 2A$
- Sum of first n natural numbers = $\frac{n(n+1)}{2}$
- Sum of first n odd numbers = n^2
- Sum of first n even numbers = $n(n+1)$

TERM SELECTION HACK:

- 3 Terms: $(a-d), a, (a+d)$ [Sum = $3a$]
- 4 Terms: $(a-3d), (a-d), (a+d), (a+3d)$ [Sum = $4a$, diff = $2d$]

14. 🧠 COMPLETE CHAPTER MINDMAP

ARITHMETIC PROGRESSION (MINDMAP HIERARCHY)

• 1. DEFINITION & BASICS

- Sequence with constant difference $d = a_k - a_{k-1}$
- Types of d : Positive (Increasing), Negative (Decreasing), Zero (Constant)

• 2. GENERAL TERM (a_n)

- From Start: $a_n = a + (n - 1)d$
- From End: $a_n = l - (n - 1)d$ (or reverse A.P.)
- Properties: $a_m - a_k = (m - k)d$

• 3. SUM OF TERMS (S_n)

- Formulas: $S_n = \frac{n}{2}[2a + (n-1)d]$ OR $S_n = \frac{n}{2}[a + l]$
- Link to Term: $a_n = S_n - S_{n-1}$

• 4. TRICKS & SHORTCUTS

- If $S_n = An^2 + Bn \rightarrow d = 2A$
- Symmetrical term selection for fast cancellation

15. ✅ CHAPTER MASTER CHECKLIST

[✓] All Concepts & Definitions Understood

[✓] Tinku Sir Tricks Memorized

[✓] General Term Formulas Practiced

[✓] 2023 - 2026 Board PYQs Solved

[✓] Sum of Terms Formulas Mastered

[✓] Top 10 Common Mistakes Avoided

[✓] Term from End Hack Mastered

[✓] One-Page Revision Completed

🎉 CONGRATULATIONS! YOU HAVE MASTERED ARITHMETIC PROGRESSION WITH ONEPATH ACADEMY & TINKU SIR! 🎉