



SUBJECT: Mathematics (Code 041)

CHAPTER: Surface Area & Volume

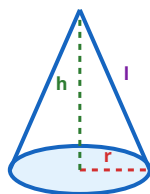
TEACHER: Tinku Sir

TARGET: 100% Board Exam Success

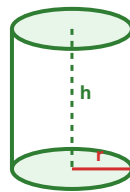
1. MASTER 3D SHAPES & BASIC FORMULA BLUEPRINT

In Class 10 CBSE, we deal with **Combinations of Solids** (joining two or more solids together) and **Conversion of Solids** (melting/recasting). Before tackling complex combined solids, you **MUST** master individual 3D figures.

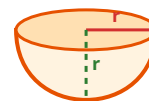
Solid Shape	Curved / Lateral Surface Area (CSA/LSA)	Total Surface Area (TSA)	Volume (V)	Key Variables & Relations
Cube	$4a^2$	$6a^2$	a^3	a = edge length Diagonal = $a\sqrt{3}$
Cuboid	$2h(l + b)$	$2(lb + bh + hl)$	$l \times b \times h$	l, b, h = dimensions Diagonal = $\sqrt{l^2 + b^2 + h^2}$
Right Circular Cylinder	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = base radius, h = height Base Area = πr^2
Right Circular Cone	πrl	$\pi r(r + l)$	$\frac{1}{3} \pi r^2 h$	$l = \sqrt{r^2 + h^2}$ (Slant Height) $h = \sqrt{l^2 - r^2}$
Sphere	$4\pi r^2$	$4\pi r^2$ (Same as CSA)	$\frac{4}{3} \pi r^3$	r = radius Diameter $d = 2r$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3} \pi r^3$	Flat top face = πr^2 $TSA = CSA + Base = 2\pi r^2 + \pi r^2$



Cone: $l = \sqrt{r^2 + h^2}$



Cylinder: $V = \pi r^2 h$



Hemisphere: $TSA = 3\pi r^2$

Figure 1.1: Core Geometric Elements of Standard 3D Solids

NEVER ADD INDIVIDUAL TSAs WHEN SOLIDS ARE COMBINED!

Imagine taking a paintbrush and painting the combined solid from the outside. *Surface area is ONLY what you can touch/paint!*

- When two solids are joined together, the overlapping contacting faces **DISAPPEAR**.
- **Formula Rule:** *Total Surface Area of Combined Solid* = Σ (*Exposed Curved/Flat Surface Areas*)
- *Example:* Capsule (Cylinder + 2 Hemispheres) = *CSA of Cylinder* + $2 \times$ (*CSA of Hemisphere*) = $2\pi rh + 2(2\pi r^2)$.

2. CONCEPT 1: SURFACE AREA OF COMBINED SOLIDS

① **WHAT?** Combining two or more basic solids (e.g., hemisphere mounted on a cube, cone surmounted on a hemisphere, cylinder with conical cavity).

② **WHY DOES IT WORK?** The concept relies on visualising the outer boundary of the resultant structure. Internal surfaces that touch each other cease to exist on the outer boundary.

③ **HOW TO APPLY?** Identify each component shape, determine which surfaces are visible/exposed, and sum their respective areas.

Standard Combination Archetypes in CBSE Exams

Combination Type	Visual Layout Description	Total Surface Area (TSA) Formula	Total Volume (V) Formula
Cone on Hemisphere (Toy / Ice-cream)	Cone base sits perfectly on the flat face of Hemisphere ($r_{\text{cone}} = r_{\text{hemi}}$)	$TSA = CSA_{\text{cone}} + CSA_{\text{hemi}} = \pi rl + 2\pi r^2$	$V = V_{\text{cone}} + V_{\text{hemi}} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$
Hemisphere on Cube Side (Decor block)	Hemisphere of max diameter l on top of cube of edge l	$TSA = \text{Surface Area of Cube} + CSA_{\text{hemi}} - \text{Base Area}_{\text{hemi}} = 6l^2 + 2\pi r^2 - \pi r^2 = 6l^2 + \pi r^2$	$V = V_{\text{cube}} + V_{\text{hemi}} = l^3 + \frac{2}{3}\pi r^3$
Cylinder with 2 Hemispherical Ends (Capsule)	Cylinder in middle, hemispherical caps at both ends	$TSA = CSA_{\text{cyl}} + 2 \times CSA_{\text{hemi}} = 2\pi rh + 4\pi r^2$	$V = V_{\text{cyl}} + 2 \times V_{\text{hemi}} = \pi r^2 h + \frac{4}{3}\pi r^3$
Conical Cavity Scooped Out of Cylinder	A cone of same height & radius carved out from a cylinder	$TSA = CSA_{\text{cyl}} + \text{Base Area}_{\text{cyl}} + CSA_{\text{cone inner}} = 2\pi rh + \pi r^2 + \pi rl$	$V_{\text{remaining}} = V_{\text{cyl}} - V_{\text{cone}} = \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

Never substitute $\pi = 22/7$ or square numbers in individual terms! Always write algebraic expressions first, take out common terms, and calculate ONLY at the very last step!

$$TSA = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

Time Saved: Reduces calculations by 60% and eliminates 90% of arithmetic errors!

SOLVED EXAMPLE 1 (NCERT / Board Standard)**CBSE 2024 VERIFIED**

Question: A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

 **CONCEPT IDENTIFICATION:** Cylinder with two hemispherical cavities scooped out.

 **GIVEN:** Height of cylinder $h = 10 \text{ cm}$, Radius $r = 3.5 \text{ cm} = \frac{7}{2} \text{ cm}$.

 **TO FIND:** Total Surface Area (TSA) of the remaining article.

 **METHOD & FORMULA:** When cavities are scooped out, new inner hemispherical surfaces are created. Therefore, total visible surface area increases!

$$TSA = \text{CSA of Cylinder} + 2 \times (\text{CSA of Hemisphere}) = 2\pi rh + 2(2\pi r^2) = 2\pi r(h + 2r)$$

 **STEP-BY-STEP SOLUTION:**

1. Substitute values in algebraic form: $r = \frac{7}{2}$, $h = 10$.
2. $TSA = 2\pi \times \frac{7}{2} \times (10 + 2 \times \frac{7}{2})$
3. Cancel common terms: $TSA = 22\pi (10 + 7) = 22\pi \times 17$
4. $TSA = 374\pi \text{ cm}^2$

 **FINAL ANSWER:** The total surface area of the article is $374\pi \text{ cm}^2$.

 **COMMON MISTAKE 1: SCOOPING OUT DOES NOT MEAN SUBTRACTING AREA!**

 **WRONG THINKING:** "Scooping out material reduces the total surface area, so subtract CSA of hemispheres."

 **CORRECT THINKING:** Scooping out material REDUCES volume, but INCREASES exposed surface area because you can now touch/paint inside the cavity!

3. CONCEPT 2: VOLUME OF COMBINED SOLIDS

① **WHAT?** Volume represents the total 3D space enclosed or capacity of a container.

② **WHY DOES IT WORK?** Volume is strictly additive and subtractive! Unlike surface area, volume depends purely on total mass/space.

③ **HOW TO APPLY?**

- If solids are attached together: $V_{\text{total}} = V_1 + V_2 + \dots$
- If a cavity/solid is removed: $V_{\text{remaining}} = V_{\text{original}} - V_{\text{removed}}$

SOLVED EXAMPLE 2 (Gulab Jamun Problem - High Frequency)

BOARD HOTS

Question: A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

GIVEN:

- Total length of gulab jamun $L = 5 \text{ cm}$
- Diameter $d = 2.8 \text{ cm} \implies r = 1.4 \text{ cm} = 7/5 \text{ cm}$
- Length of cylindrical part $h = L - 2r = 5 - 2(1.4) = 5 - 2.8 = 2.2 \text{ cm}$

STEP-BY-STEP SOLUTION:

1. Volume of 1 Gulab Jamun:

$$V_1 = V_{\text{cyl}} + 2 \times V_{\text{hemi}} = \pi r^2 h + \frac{4}{3} \pi r^3 = \pi r^2 \left(h + \frac{4}{3} r \right)$$

2. Substitute $r = 1.4$ and $h = 2.2$:

$$V_1 = \frac{22}{7} \times (1.4)^2 \times \left(2.2 + \frac{4}{3} \times 1.4 \right) = 22 \times 0.2 \times 1.4 \times \left(2.2 + \frac{5.6}{3} \right) = 6.16 \times \left(\frac{6.6 + 5.6}{3} \right) = 6.16 \times \frac{12.2}{3} = \frac{75.152}{3} \text{ cm}^3$$

3. Total Volume of 45 Gulab Jamuns:

$$V_{45} = 45 \times \frac{75.152}{3} = 15 \times 75.152 = 1127.28 \text{ cm}^3$$

4. Volume of Syrup (30%):

$$V_{\text{syrup}} = 30\% \times 1127.28 = \frac{30}{100} \times 1127.28 = 338.184 \text{ cm}^3 \approx 338 \text{ cm}^3$$

 **FINAL ANSWER:** Approximate volume of syrup is 338 cm^3 .

TINKU SIR TRICK 2: The "EMBEDDED HEIGHT" Shortcut

In problems with hemispherical caps on cylinders or cones on hemispheres:

Height of Cylindrical Part (h) = Total Height (H) - (Radius of End 1 + Radius of End 2)

Why? The height of a hemisphere is always equal to its **RADIUS (r)**! Memorize this: $h_{\text{hemi}} = r$.

4. QUESTION IDENTIFICATION MATRIX ("QUESTION DEKHO → TYPE PEHCHANO")

Train your mind to immediately map keywords in CBSE board exam questions to the correct formula and strategy!

Question Clue / Keywords	Concept & Solid Type	Required Formula / Mathematical Approach
"Surmounted on", "Mounted on"	Combination of Solids (Area)	$TSA = \text{Sum of exposed CSAs}$ (Check for overlap)
"Scooped out", "Carved out", "Drilled"	Cavity Problem	$\text{Area} = \text{Outer Area} + \text{Inner Cavity CSA}$ $\text{Volume} = \text{Original Volume} - \text{Removed Volume}$
"Melting", "Recasting", "Reshaped"	Conversion of Solids	Equate Volumes: $V_{\text{Initial Solid}} = V_{\text{Final Solid}}$
"Flow through pipe", "Rate of flow"	Speed-Volume Pipeline	$\text{Length of water column } (h) = \text{Speed} \times \text{Time}$
"Embankment", "Well dug out"	Hollow Cylinder / Ring	$V_{\text{earth dug}} = \pi r_{\text{well}}^2 h_{\text{well}} = \pi (R^2 - r^2) h_{\text{embankment}}$

Question Clue / Keywords	Concept & Solid Type	Required Formula / Mathematical Approach
"How many small spheres/cubes..."	Number of Objects	$n = \frac{V_{\text{larger solid}}}{V_{\text{one small solid}}}$

5. AUTHENTIC CBSE BOARD PYQS & BOARD PATTERNS (2023–2026)

VERIFIED PYQ 1 (3 Marks)

CBSE 2023 BOARD

Question: A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. (Take $\pi = 3.14$)

SOLUTION (CBSE Marking Scheme Format):

- Radius of cone and hemisphere $r = \frac{4}{2} = 2 \text{ cm}$ [1/2 Mark]
- Height of cone $h = 2 \text{ cm}$
- $V_{\text{toy}} = V_{\text{cone}} + V_{\text{hemi}} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ [1 Mark]
- $V_{\text{toy}} = \frac{1}{3}\pi r^2 (h + 2r)$
- $V_{\text{toy}} = \frac{1}{3} \times 3.14 \times (2)^2 \times (2 + 2(2))$ [1 Mark]
- $V_{\text{toy}} = \frac{1}{3} \times 3.14 \times 4 \times 6 = 3.14 \times 8 = 25.12 \text{ cm}^3$ [1/2 Mark]

 **FINAL ANSWER:** Volume of the toy = 25.12 cm^3 .

VERIFIED PYQ 2 (5 Marks)

CBSE 2024 BOARD

Question: A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹500 per m^2 . (Note that the base of the tent will not be covered with canvas).

SOLUTION:

- For Cylindrical part: Radius $r = 2 \text{ m}$, Height $h = 2.1 \text{ m}$
- For Conical part: Radius $r = 2 \text{ m}$, Slant height $l = 2.8 \text{ m}$
- $\text{Area of Canvas} = \text{CSA of Cylinder} + \text{CSA of Cone}$ [1 Mark]
- $\text{Area} = 2\pi rh + \pi rl = \pi r(2h + l)$ [1 Mark]
- $\text{Area} = \frac{22}{7} \times 2 \times [2(2.1) + 2.8] = \frac{44}{7} \times [4.2 + 2.8] = \frac{44}{7} \times 7 = 44 \text{ m}^2$ [1.5 Marks]
- $\text{Cost of Canvas} = 44 \times 500 = \text{₹ } 22,000$ [1.5 Marks]

 **FINAL ANSWER:** Area of canvas = 44 m^2 , Total Cost = ₹22,000.

Scenario: An architectural student designed a building model comprising a central cylinder with a dome shaped as a hemisphere on top. The total height of the building model is 30 cm and the base diameter is 14 cm.

Questions:

- (i) What is the height of the cylindrical portion? [1 Mark]
- (ii) Find the total inner volume of the model. [2 Marks]
- (iii) Find the ratio of the volume of the cylindrical part to the hemispherical part. [1 Mark]

 SOLUTION:

- (i) Base diameter = 14 cm $\implies r = 7 \text{ cm}$.
Height of hemisphere = $r = 7 \text{ cm}$.
Height of cylinder $h = 30 - 7 = 23 \text{ cm}$.
- (ii) $V_{\text{total}} = \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 \left(h + \frac{2}{3}r \right) = \frac{22}{7} \times 49 \times \left(23 + \frac{14}{3} \right) = 154 \times \left(\frac{83}{3} \right) = \frac{12782}{3} = 4260.67 \text{ cm}^3$.
- (iii) $\text{Ratio} = \frac{V_{\text{cyl}}}{V_{\text{hemi}}} = \frac{\pi r^2 h}{\frac{2}{3}\pi r^3} = \frac{3h}{2r} = \frac{3(23)}{2(7)} = \frac{69}{14} = 69:14$.

6. CONCEPT-WISE PRACTICE SET (BASIC TO HOTS)

LEVEL 1: BASIC (Direct Formula Application)

1. Two cubes each of volume 64 cm^3 are joined end to end. Find the total surface area of the resulting cuboid. [Ans: 160 cm^2]
2. Find the slant height of a cone whose radius is 5 cm and vertical height is 12 cm. [Ans: 13 cm]

LEVEL 2: APPLICATION (Standard Board Questions)

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π . [Ans: $\pi \text{ cm}^3$]
2. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. Find its volume. [Ans: 346.51 cm^3]

LEVEL 3: ADVANCED & HOTS

1. A juice seller was serving his customers using glasses as shown. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. (Use $\pi = 3.14$) [Ans: Apparent = 196.25 cm^3 , Actual = 163.54 cm^3]

7. VISUAL CHAPTER MIND MAP (2-MINUTE REVISION)

SURFACE AREA & VOLUME (CLASS 10)

- 1. COMBINATION OF SOLIDS
 - Surface Area: SUM OF EXPOSED SURFACES ONLY (Ignore hidden overlaps!)
 - Toy (Cone + Hemisphere) $\rightarrow$ TSA = $\pi r l + 2\pi r^2$
 - Capsule (Cyl + 2 Hemispheres) $\rightarrow$ TSA = $2\pi r h + 4\pi r^2$
 - Cube + Hemisphere Base $\rightarrow$ TSA = $6a^2 + \pi r^2$
 - Volume: STRICTLY ADDITIVE ($V_{\text{total}} = V_1 + V_2$)
- 2. CAVITY & SCOOPING PROBLEMS
 - Surface Area: INCREASES! (TSA = Original Area + Inner Cavity Area)
 - Volume: DECREASES! ($V_{\text{remaining}} = V_{\text{original}} - V_{\text{scooped}}$)
- 3. CONVERSION OF SOLIDS
 - Rule: Volume remains CONSTANT ($V_{\text{initial}} = V_{\text{final}}$)
 - Number of Recast Objects = (Volume of Large Solid) / (Volume of 1 Small Solid)
- GOLDEN CALCULATION TRICKS
 - 1. Keep ' π ' in symbol until the final line!
 - 2. Factorise expressions (e.g., $2\pi r(h + r)$) before numerical substitution!
 - 3. Height of Hemisphere is ALWAYS equal to its RADIUS ($h = r$)!

8. ONEPATH ONE-PAGE HIGH-YIELD REVISION SHEET

LAST MINUTE BOARD FORMULA CHEAT SHEET

Cone Slant Height: $l = \sqrt{r^2 + h^2}$	Cone CSA: $\pi r l$	Cone TSA: $\pi r(r + l)$
Cylinder CSA: $2\pi r h$	Cylinder TSA: $2\pi r(r + h)$	Cylinder Vol: $\pi r^2 h$
Sphere Area: $4\pi r^2$	Hemisphere CSA: $2\pi r^2$	Hemisphere TSA: $3\pi r^2$
Sphere Vol: $\frac{4}{3}\pi r^3$	Hemisphere Vol: $\frac{2}{3}\pi r^3$	Cone Vol: $\frac{1}{3}\pi r^2 h$

★ Key Constant Values: π approx $\frac{22}{7}$ approx 3.14159 | $\sqrt{2}$ approx 1.414 | $\sqrt{3}$ approx 1.732 | $\sqrt{5}$ approx 2.236

9. ⚠ COMMON MISTAKES & BOARD PRESENTATION STRATEGY

✗ Common Student Mistake	✓ Correct CBSE Approach	💡 How to Avoid / Tinku Sir Tip
Writing wrong units like cm^2 for Volume or cm^3 for Area.	Area is always in cm^2 / m^2 . Volume is always in cm^3 / m^3 .	Write unit explicitly next to EVERY final line. Half mark is deducted for wrong/missing units!
Using $\pi = \frac{22}{7}$ when question explicitly states "Take $\pi = 3.14$ ".	Always read the bracket instruction at the end of the question first!	Highlight or underline $\pi = 3.14$ in reading time.
Adding TSAs directly when two solids are joined.	Add only the exposed CSAs + uncovered base areas.	Use the "Paintbrush Rule" — paint only what is visible!
Confusing diameter with radius.	Convert diameter to radius immediately: $r = \frac{d}{2}$.	Write $r = \text{ext}\{\text{value}\}$ in the very first line of your step-by-step solution.

10. FINAL CHAPTER CHECKLIST FOR BOARD EXAM

- [✓] All standard 3D solid formulas memorized
- [✓] Combination of Cone + Hemisphere mastered
- [✓] Cylinder + Hemispheres (Capsule) mastered
- [✓] Scooped-out cavity problems practiced
- [✓] "Paintbrush Rule" applied for surface areas
- [✓] Factorisation technique used to simplify calculations
- [✓] Authentic 2023-2026 PYQs solved
- [✓] Competency-based case study questions practiced
- [✓] Units written correctly (cm^2 vs cm^3)
- [✓] Final revision sheet reviewed

 **DESIGNED BY TINKU SIR & ONEPATH ACADEMY TEAM — SCORE 100/100 IN CBSE MATHS!** 