



1. CHAPTER OVERVIEW & MAP

Welcome to **Real Numbers**! As per the latest CBSE syllabus, this chapter carries **6 Marks** in your Board Examinations. It lays the foundational bedrock for high school algebra and number theory.

REAL NUMBERS ($\mathbb{R}$)

- Fundamental Theorem of Arithmetic (FTA)
 - Prime Factorisation (Unique Factorisation)
 - Finding HCF & LCM using Prime Factors
 - Formula: $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$
- Irrational Numbers & Proofs of Irrationality
 - Definition & Properties (Non-terminating Non-repeating)
 - Fundamental Lemma: If $p \mid a^2$, then $p \mid a$
 - Contradiction Proofs ($\sqrt{2}$, $\sqrt{3}$, $3 + 2\sqrt{5}$, etc.)
- Board Exam Question Patterns
 - Word Problems (HCF vs LCM Identification)
 - Proof of Irrationality (3-Mark Guarantee)
 - Application of FTA in divisibility & unit digits

2. CONCEPT 1: THE FUNDAMENTAL THEOREM OF ARITHMETIC (FTA)

① **WHAT?** The Fundamental Theorem of Arithmetic states that every composite number can be uniquely expressed as a product of prime numbers, regardless of the order in which the prime factors occur.

② **WHY?** Primes are the "building blocks" (atoms) of all integers. Just as every chemical compound has a unique chemical formula, every composite integer has a unique "prime signature".

③ **HOW?** We break a number down into its prime factors using repeated division or a prime factor tree.



THEOREM STATEMENT (NCERT 1.1)

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

$$x = p_1^{a_1} \times p_2^{a_2} \times \dots \times p_n^{a_n}$$

where $p_1, p_2, \dots, p_n$ are distinct primes in ascending order, and $a_1, a_2, \dots, a_n$ are natural numbers.



TINKU SIR TRICK: "PRIME ATOM RULE"

Do not stop factorising until EVERY branch end is a pure prime! Always write primes in **ASCENDING ORDER** with exponential powers. This prevents missed factors during HCF/LCM calculations.



SOLVED EXAMPLE 1 (PRIME FACTORISATION)

Question: Express 32760 as a product of its prime factors.

Step-by-step Solution:

1. Divide by 2: $32760 \div 2 = 16380$
2. Divide by 2: $16380 \div 2 = 8190$
3. Divide by 2: $8190 \div 2 = 4095$
4. Divide by 3: $4095 \div 3 = 1365$
5. Divide by 3: $1365 \div 3 = 455$
6. Divide by 5: $455 \div 5 = 91$
7. Divide by 7: $91 \div 7 = 13$ (Prime)

Final Answer: $32760 = 2^3 \times 3^2 \times 5^1 \times 7^1 \times 13^1$

3. CONCEPT 2: HCF & LCM USING PRIME FACTORISATION

MASTER FORMULAS: HCF & LCM

- **HCF (Highest Common Factor):** Product of the smallest power of each *common* prime factor involved in the numbers.
- **LCM (Lowest Common Multiple):** Product of the greatest power of *each* prime factor involved in the numbers.
- **FUNDAMENTAL RELATIONSHIP (For 2 numbers only!):**

$$HCF(a, b) \times LCM(a, b) = a \times b$$

WARNING / COMMON MISTAKE

CRITICAL TRAP: The formula $HCF(a, b, c) \times LCM(a, b, c) = a \times b \times c$ is **WRONG for three numbers!** It applies *ONLY* to two positive integers. For 3 numbers, use prime powers directly.

ONEPATH HACK: POWER RULE TABLE

To find HCF and LCM instantly without confusion, create a power matrix:

Number	2	3	5
$a = 12 = 2^2 \times 3^1$	2^2	3^1	5^0
$b = 18 = 2^1 \times 3^2$	2^1	3^2	5^0
HCF (Take MIN Power)	2^1	3^1	-
LCM (Take MAX Power)	2^2	3^2	-

$$HCF = 2^1 \times 3^1 = 6 \mid LCM = 2^2 \times 3^2 = 36$$

4. QUESTION IDENTIFICATION SYSTEM: WORD PROBLEMS

In CBSE Board exams, students often get confused between when to compute **HCF** and when to compute **LCM**. Use this rule book:

Question Clue / Keywords	Concept To Use	Reason / Logic
"Maximum capacity", "Largest size", "Greatest number", "Divide into equal groups", "Maximum stacks"	HCF	You need a number smaller than or equal to given inputs that divides them completely.
"Tolls together", "Bells tolling simultaneously", "Traffic lights changing together", "Minimum distance", "Find next time they meet"	LCM	You need a future/larger common quantity that is a multiple of all individual cycle times.

SOLVED EXAMPLE 2 (CBSE BOARD STYLE WORD PROBLEM)

Question: Three bells toll at intervals of 9, 12, and 15 minutes respectively. If they start tolling together, after what minimum time will they toll together next?

 **Concept Identification:** Keywords are "minimum time" and "toll together next". This requires finding the **LCM**.

 **Given:** Intervals $t_1 = 9$, $t_2 = 12$, $t_3 = 15$.

 **Step-by-Step Solution:**

1. $9 = 3^2$

2. $12 = 2^2 \times 3^1$

3. $15 = 3^1 \times 5^1$

4. Take maximum power of each prime factor involved ($\{2, 3, 5\}$):

$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5^1 = 4 \times 9 \times 5 = 180 \text{ minutes.}$$

 **Final Answer:** They will toll together after **180 minutes (3 hours)**.

5. CONCEPT 3: PROOF OF IRRATIONALITY

① **WHAT?** An irrational number cannot be expressed in the form p/q , where $p, q \in \mathbb{Z}$ and $q \neq 0$.

② **FUNDAMENTAL LEMMA (Theorem 1.2):** Let p be a prime number. If p divides a^2 (where a is a positive integer), then p divides a .

TINKU SIR TRICK: "CONTRADICTION BLUEPRINT"

Every single irrationality proof in CBSE follows 5 fixed steps. Memorize this exact template:

1. **Assume Contrary:** Let the given number be rational = a/b (co-prime, $b \neq 0$).
2. **Square & Rearrange:** Isolate the root and square both sides.
3. **Apply Lemma:** Show that prime p divides $a^2 \implies p$ divides a (Let $a = pc$).
4. **Substitute Back:** Show that prime p also divides b .
5. **Declare Contradiction:** a and b have common factor p , contradicting co-primality! Hence irrational.

THEOREM PROOF: PROVE THAT $\sqrt{5}$ IS IRRATIONAL (3-MARK CBSE CLASSIC)

Proof:

Let us assume, to the contrary, that $\sqrt{5}$ is rational.

Therefore, we can find co-prime integers a and b ($b \neq 0$) such that:

$$\sqrt{5} = a / b$$

Squaring both sides:

$$5 = a^2 / b^2 \implies 5b^2 = a^2 \text{ --- (Equation 1)}$$

This means 5 divides a^2 . By Theorem 1.2, **5 divides a** .

So, we can write $a = 5c$ for some integer c .

Substituting $a = 5c$ in Equation 1:

$$5b^2 = (5c)^2 \implies 5b^2 = 25c^2 \implies b^2 = 5c^2$$

This means 5 divides b^2 . By Theorem 1.2, **5 divides b** .

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b are co-prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is rational.

Hence, $\sqrt{5}$ is irrational.

[Q.E.D.]

SOLVED EXAMPLE 3 (COMPOSITE IRRATIONALITY)

Question: Prove that $3 + 2\sqrt{5}$ is irrational, given that $\sqrt{5}$ is irrational.

Solution:

Let us assume to the contrary that $3 + 2\sqrt{5}$ is rational.

Then $3 + 2\sqrt{5} = a/b$, where $a, b \in \mathbb{Z}$, $b \neq 0$, and a, b are co-prime.

Rearranging the equation to isolate $\sqrt{5}$:

$$2\sqrt{5} = (a/b) - 3 \implies 2\sqrt{5} = (a - 3b) / b \implies \sqrt{5} = (a - 3b) / (2b)$$

Since a and b are integers, $(a - 3b) / (2b)$ is a rational number.

This implies that $\sqrt{5}$ is rational.

But this contradicts the given fact that $\sqrt{5}$ is irrational.

Hence, our assumption was wrong. $3 + 2\sqrt{5}$ **is irrational**.

6. COMPARISON TABLE: RATIONAL VS IRRATIONAL

Property	Rational Numbers ($\mathbb{Q}$)	Irrational Numbers ($\mathbb{Q}'$)
Form	Can be written as p/q ($q \neq 0, p, q \in \mathbb{Z}$)	Cannot be written as p/q
Decimal Expansion	Either <i>Terminating</i> or <i>Non-terminating Recurring</i>	<i>Non-terminating Non-recurring</i>
Examples	$3/5, 0.75, 4.333..., -22/7$	$\sqrt{2}, \sqrt{3}, \pi, 0.1010010001...$
Sum / Difference with Rational	Rational $\pm$ Rational = Rational	Rational $\pm$ Irrational = Irrational

Property	Rational Numbers ($\mathbb{Q}$)	Irrational Numbers ($\mathbb{Q}'$)
Product / Quotient with Non-zero Rational	Rational $\times$ Rational = Rational	Non-zero Rational $\times$ Irrational = Irrational

7. VERIFIED CBSE BOARD PYQS (2023 - 2026)

VERIFIED CBSE PYQ 2024 (1 Mark MCQ)

Question: If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then find $HCF(a, b)$.

Solution:

- $a = x^3y^2$
- $b = x^1y^3$
- To find HCF, take the minimum power of common prime bases x and y :
- Power of $x = \min(3, 1) = 1$
- Power of $y = \min(2, 3) = 2$

Answer: $HCF(a, b) = xy^2$

VERIFIED CBSE PYQ 2023 (2 Marks)

Question: Explain why $7 \times 11 \times 13 + 13$ is a composite number.

Solution:

Given expression: $E = 7 \times 11 \times 13 + 13$

Taking 13 common:

$$E = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1) = 13 \times 78$$

$$E = 13 \times (2 \times 3 \times 13) = 2 \times 3 \times 13^2$$

Since the given number can be expressed as a product of prime factors other than 1 and itself, by the **Fundamental Theorem of Arithmetic**, it is a **composite number**.

VERIFIED CBSE PYQ 2025 (3 Marks)

Question: Find HCF and LCM of 404 and 96 and verify that $HCF \times LCM = \text{Product of two numbers}$.

Solution:

1. Prime Factorisation:

$$404 = 2^2 \times 101$$

$$96 = 2^5 \times 3^1$$

2. $HCF(404, 96) = 2^2 = 4$

3. $LCM(404, 96) = 2^5 \times 3^1 \times 101^1 = 32 \times 3 \times 101 = 9696$

4. Verification:

$$\text{LHS} = \text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

$$\text{RHS} = \text{Product of numbers} = 404 \times 96 = 38784$$

LHS = RHS. **Verified!**

8. CONCEPT-WISE PRACTICE SET

LEVEL 1 — BASIC (1 Mark)

1. **BASIC** What is the exponent of 2 in the prime factorisation of 144? [Ans: 4]
2. **BASIC** If $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$. [Ans: 22338]

LEVEL 2 — APPLICATION (2 & 3 Marks)

1. **MODERATE** Check whether 6^n can end with the digit 0 for any natural number n .
[Hint: Prime factors of 6 are 2 and 3. For a number to end in 0, prime factorisation must contain 2 and 5. Since 5 is absent, 6^n can never end with 0.]
2. **MODERATE** Prove that $2 - 3\sqrt{5}$ is an irrational number.

LEVEL 3 — HOTS & COMPETENCY (4 & 5 Marks)

1. **HOTS** A rectangular courtyard 18 m 72 cm long and 13 m 20 cm broad is to be paved with square tiles of the same size. Find the least possible number of such tiles.
[Solution: Convert dimensions to cm: 1872 cm and 1320 cm. Side of largest square tile = $\text{HCF}(1872, 1320) = 24$ cm.
Number of tiles = $(1872 \times 1320) / (24 \times 24) = 78 \times 55 = 4290$ tiles.]

9. ONEPATH ONE-PAGE REVISION SHEET

LAST-MINUTE EXAM RECAP SHEET

- ☑ **FTA Statement:** Every composite number = unique product of primes.
- ☑ **HCF Formula:** Product of LOWEST powers of COMMON prime factors.
- ☑ **LCM Formula:** Product of HIGHEST powers of ALL prime factors.
- ☑ **2-Number Relation:** $a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b)$.
- ☑ **Ending in Zero condition:** Prime factorisation MUST contain prime factors $2^m \times 5^n$.
- ☑ **Irrationality Lemma:** If $p \mid a^2$ (where p is prime), then $p \mid a$.
- ☑ **Proof Technique:** Always use Proof by Contradiction assuming a/b co-prime.

10. ⚠️ STUDENTS OFTEN MAKE THESE MISTAKES

❌ WRONG APPROACH VS ✅ CORRECT APPROACH

- ❌ **Wrong:** Writing $HCF(a, b, c) \times LCM(a, b, c) = a \times b \times c$ for 3 numbers.
✅ **Correct:** Apply product relation strictly for 2 numbers. For 3 numbers, compute independently using prime exponent matrices.
- ❌ **Wrong:** Forgetting to state that a and b are *co-prime* in irrationality proofs.
✅ **Correct:** Explicitly state: "where a and b are co-prime integers and $b \neq 0$ ". You lose 0.5 marks without this line!
- ❌ **Wrong:** Writing HCF when asked for time cycles (bells/lights).
✅ **Correct:** Cycles repeating together always require **LCM**.

11. 🏆 HOW TO SCORE FULL MARKS (BOARD STRATEGY)

- **Step Marking Secret:** Show full prime factor trees or prime division steps in rough/main working column. Don't write direct answers for 2 or 3 mark questions.
- **Presentation:** Always box your final answer: e.g., $HCF = 12$.
- **Time Management:** Real numbers section should take no more than **10–12 minutes** in the board exam!

12. 🏁 FINAL CHAPTER CHECKLIST

- ☑ Fundamental Theorem of Arithmetic clear & memorized
- ☑ Prime factorisation of large numbers practiced
- ☑ HCF & LCM power rule understood
- ☑ Formula $a \times b = HCF \times LCM$ applied correctly
- ☑ 3-Mark Proof of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ written 2 times by hand
- ☑ Proof of $a + b\sqrt{c}$ form mastered
- ☑ Word problems HCF vs LCM distinguished
- ☑ All verified PYQs (2023-2026) solved