

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

CBSE 2026-27 EDITION

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Class 10 Mathematics • Complete Master Notes & Board Strategy

Teacher: Tinku Sir **Brand:** OnePath Academy **Target:** CBSE Board Exam (100/100) **Focus:** NCERT + PYQs + Competency

1. CHAPTER OVERVIEW & BOARD WEIGHTAGE

This chapter carries a weightage of **6 to 8 marks** in the CBSE Class 10 Board Examination. Questions typically range from 1-mark MCQs, 2-mark & 3-mark algebraic solutions, to 5-mark word problems or 4-mark case-study questions.

2. CORE CONCEPTS & ALGEBRAIC STRUCTURE

Concept 1: General Form of Pair of Linear Equations

A pair of linear equations in two variables x and y is written as:

STANDARD GENERAL FORM

$$a_1x + b_1y + c_1 = 0 \quad (\text{Eq. 1})$$

$$a_2x + b_2y + c_2 = 0 \quad (\text{Eq. 2})$$

Conditions: $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers, and $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.

Geometrical Meaning: Every linear equation in two variables represents a **straight line** on a Cartesian plane. A system of two linear equations represents **two straight lines**, whose relative positioning determines the nature of solutions.

Concept 2: Graphical Conditions for Consistency & Solutions

The relationship between the coefficients dictates whether the lines intersect, are parallel, or coincide.

Ratio Comparison	Graphical Representation	Algebraic Interpretation	Consistency
$a_1/a_2 \neq b_1/b_2$	Intersecting Lines	Unique Solution (1 point)	Consistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident / Overlapping Lines	Infinitely Many Solutions	Dependent / Consistent
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel Lines	No Solution (0 points)	Inconsistent

🔥 TINKU SIR TRICK — RATIO CHECKER IN 5 SECONDS!

Before solving ANY linear equation algebraically, ALWAYS write ratios in standard form $a_1x + b_1y + c_1 = 0$!

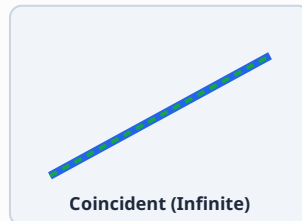
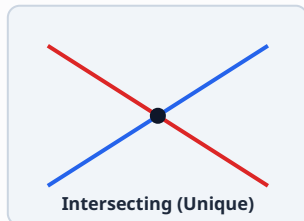
- **Unequal slopes?** ($a_1/a_2 \neq b_1/b_2$) → Cross lines → 1 Unique Point!
- **Everything equal?** ($a_1/a_2 = b_1/b_2 = c_1/c_2$) → Twin lines → Infinite Points!
- **Ratio fails at constant?** ($a_1/a_2 = b_1/b_2 \neq c_1/c_2$) → Railway tracks (Parallel) → NO Solution!

COMMON MISTAKE ALERT!

Mistake: Forgetting to bring constants c_1 and c_2 to the SAME side of the equation before taking ratios.

Example: If given $2x + 3y = 7$ and $4x + 6y + 14 = 0$, writing $c_1/c_2 = 7/14 = 1/2$ is WRONG! First convert both to standard form: $2x + 3y - 7 = 0$, so $c_1/c_2 = -7/14 = -1/2$.

Visual Representation of Line Conditions



3. ALGEBRAIC METHODS OF SOLVING

Method 1: Substitution Method

Step 1: Express one variable in terms of the other from one equation (preferably equation with coefficient 1 or -1).

Step 2: Substitute this expression into the OTHER equation to obtain a single-variable linear equation.

Step 3: Solve for that variable.

Step 4: Back-substitute to find the second variable.

Method 2: Elimination Method (Most Preferred in Board Exams)

Step 1: Multiply one or both equations by non-zero constants so that coefficients of one variable become equal in absolute value.

Step 2: Add or subtract the equations to eliminate that variable.

Step 3: Solve the resulting linear equation in one variable.

Step 4: Substitute back to find the remaining variable.



ONEPATH HACK — SPECIAL SYMMETRIC EQUATIONS (LARGE COEFFICIENTS)

When coefficients of x and y are interchanged in the two equations:

Form: $ax + by = c$ and $bx + ay = d$ (e.g., $99x + 101y = 499$, $101x + 99y = 501$)

DO NOT MULTIPLY LARGE NUMBERS! Use the ADD-SUBTRACT Hack:

- ADD Eq 1 and Eq 2:** Gives $(a+b)x + (a+b)y = (c+d) \rightarrow x + y = K_1$
- SUBTRACT Eq 1 and Eq 2:** Gives $(a-b)x - (a-b)y = (c-d) \rightarrow x - y = K_2$
- Solve the simplified equations $x + y = K_1$ and $x - y = K_2$ in 5 seconds!



4. MASTER WORD PROBLEMS & QUESTION IDENTIFICATION

QUESTION DEKHO → TYPE PECHANO → METHOD CHUNO

Word Problem Type	Key Clues / Keywords	Standard Variable Assumption & Formula
1. Age Problems	"years ago", "hence", "times as old as"	Let Present Age = x yrs, y yrs. n yrs ago = $(x - n)$; n yrs hence = $(x + n)$.
2. Two-Digit Number	"digits sum", "reversing the digits"	Ten's digit = x , Unit's digit = y . Original Number = $10x + y$; Reversed = $10y + x$.
3. Fraction Problems	"numerator", "denominator", "fraction becomes"	Let Fraction = x/y . Apply conditions on $(x + a)/(y + b)$ as specified.
4. Fixed & Variable Charge	"taxi fare", "library fee", "fixed charge"	Fixed Charge = ₹ x , Charge per km/day = ₹ y . Total = $x + ny$ = Total Fare .
5. Upstream / Downstream	"boat", "stream", "still water", "current"	Speed of Boat = x km/h, Speed of Stream = y km/h. Downstream = $(x + y)$; Upstream = $(x - y)$.

🔥 TINKU SIR TRICK — UPSTREAM VS DOWNSTREAM NEVER CONFUSE!

- **DOWNSTREAM:** Stream helps you → Push from behind → **ADD SPEEDS** $(x + y)$ (Faster!)
- **UPSTREAM:** Stream fights against you → Opposing force → **SUBTRACT SPEEDS** $(x - y)$ (Slower!)

Golden Rule: Always Boat Speed $x > y$ Stream Speed! Distance = Speed $\times$ Time.

🖋️ 5. FULLY SOLVED BOARD EXAMPLES

EXAMPLE 1: Finding Value of k for Infinite Solutions (CBSE Board Standard)

Question: Find the value of k for which the pair of equations $kx + 3y - (k - 3) = 0$ and $12x + ky - k = 0$ has infinitely many solutions.

Solution:

Step 1 (Write Coefficients):

$$a_1 = k, b_1 = 3, c_1 = -(k - 3)$$

$$a_2 = 12, b_2 = k, c_2 = -k$$

Step 2 (Condition for Infinitely Many Solutions):

$$a_1/a_2 = b_1/b_2 = c_1/c_2 \rightarrow k/12 = 3/k = -(k - 3)/(-k)$$

Step 3 (Solve First Two Ratios):

$$k/12 = 3/k \rightarrow k^2 = 36 \rightarrow k = \pm 6$$

Step 4 (Solve Second and Third Ratios to Verify):

$$3/k = (k - 3)/k \rightarrow 3k = k(k - 3)$$

$$\text{Since } k \neq 0, \text{ divide by } k: 3 = k - 3 \rightarrow k = 6.$$

Conclusion: $k = 6$ satisfies ALL three ratios simultaneously. (Note: $k = -6$ fails the constant ratio check!).

Final Answer: $k = 6$

EXAMPLE 2: Two-Digit Number Word Problem

Question: The sum of a two-digit number and the number obtained by reversing the order of its digits is 66. If the digits differ by 2, find the number. How many such numbers are there?

Solution:

Step 1: Let the ten's digit be x and unit's digit be y .

Original Number = $10x + y$. Reversed Number = $10y + x$.

Step 2 (Condition 1):

$$(10x + y) + (10y + x) = 66 \rightarrow 11x + 11y = 66 \rightarrow x + y = 6 \quad (\text{Eq. 1})$$

Step 3 (Condition 2):

Given digits differ by 2: Either $x - y = 2$ or $y - x = 2$.

Case I: $x - y = 2$

Adding Eq. 1 and Case I: $2x = 8 \rightarrow x = 4$. Then $y = 2$.

Number = $10(4) + 2 = 42$.

Case II: $y - x = 2$

Adding Eq. 1 and Case II: $2y = 8 \rightarrow y = 4$. Then $x = 2$.

Number = $10(2) + 4 = 24$.

Final Answer: The numbers are **42** and **24**. There are **2 such numbers**.

6. VERIFIED CBSE BOARD PYQS (2023 - 2026)

CBSE 2024 (3 Marks) VERIFIED BOARD PYQ

Question: Solve the following system of linear equations graphically:

$2x + y = 6$ and $2x - y + 2 = 0$. Also find the coordinates of the vertices of the triangle formed by these lines and the x-axis.

Answer Outline: Plotting lines gives intersection point $A(1, 4)$. Line 1 cuts x-axis at $B(3, 0)$ and Line 2 cuts x-axis at $C(-1, 0)$.

Vertices of triangle = $(1, 4)$, $(3, 0)$, $(-1, 0)$. Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 4 = 8 \text{ sq. units}$.

CBSE 2023 (2 Marks) VERIFIED BOARD PYQ

Question: Find whether the pair of linear equations $3x + 2y = 8$ and $6x + 4y = 9$ is consistent or inconsistent.

Solution: Ratios: $a_1/a_2 = 3/6 = 1/2$, $b_1/b_2 = 2/4 = 1/2$, $c_1/c_2 = -8/-9 = 8/9$.

Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, the lines are parallel. Hence, the system is **INCONSISTENT** (No solution).

CBSE 2025 Board Model (4 Marks) COMPETENCY / CASE STUDY

Scenario: A coaching institute charges a fixed admission fee plus a monthly tuition fee. Student A pays ₹4200 for 4 months. Student B pays ₹6000 for 7 months.

Questions: (i) Formulate linear equations. (ii) Find fixed admission fee and monthly fee.

Solution: Let Fixed Fee = ₹ x , Monthly Fee = ₹ y .

Eq 1: $x + 4y = 4200$; Eq 2: $x + 7y = 6000$.

Subtracting Eq 1 from Eq 2: $3y = 1800 \rightarrow y = 600$.

Substituting $y = 600$ in Eq 1: $x + 2400 = 4200 \rightarrow x = 1800$.

Fixed Fee = ₹1800, Monthly Fee = ₹600.

7. STUDENTS OFTEN MAKE THESE MISTAKES

Common Error	✗ Wrong Approach	✓ Correct Approach
Sign Error in Ratio Check	Taking $c_1 = 7$ for $2x + 3y = 7$ and $c_2 = 14$ for $4x + 6y + 14 = 0$	Move all constants to LHS first: $c_1 = -7$, $c_2 = 14$. Ratios must be sign-consistent!
Upstream / Downstream	Writing Upstream speed as $(y - x)$ or Stream speed minus Boat speed	Boat speed is ALWAYS greater than stream speed! Upstream = $(x - y)$, Downstream = $(x + y)$.
Two-Digit Number Form	Writing number as $x + y$ or xy	Two-digit number with ten's digit x and unit digit y is ALWAYS $10x + y$!
Quadratic Roots in Ratios	Taking $k^2 = 36 \rightarrow k = 6$ only	$k^2 = 36 \rightarrow k = \pm 6$. Must test BOTH values in the third ratio to reject extraneous solutions!

8. CHAPTER CONCEPT MIND MAP

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES ($a_1x + b_1y + c_1 = 0$)

1. RATIO CONDITIONS (Consistency)

- $a_1/a_2 \neq b_1/b_2$ → Intersecting Lines → Unique Solution (Consistent)
- $a_1/a_2 = b_1/b_2 = c_1/c_2$ → Coincident Lines → Infinite Solutions (Dependent)
- $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ → Parallel Lines → No Solution (Inconsistent)

2. SOLVING METHODS

- Algebraic:** Substitution Method & Elimination Method
- Symmetric Hack:** Add both & Subtract both when coefficients swap!
- Graphical:** Plot tables → Intersection point (x, y) = Solution

3. STANDARD WORD PROBLEM TEMPLATES

- Numbers & Digits:** Number = $10x + y$ | Reversed = $10y + x$
- Upstream / Downstream:** Down = $(x + y)$ | Up = $(x - y)$
- Fractions:** Let Fraction = x/y
- Fixed + Variable Charges:** Total = $x + ny$

9. ONEPATH ONE-PAGE REVISION SHEET

🏆 LAST-MINUTE FORMULA & TRICK CHEAT SHEET

- **Standard Form:** $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
- **Unique Solution:** $a_1/a_2 \neq b_1/b_2$ (Lines intersect at 1 point)
- **Infinite Solutions:** $a_1/a_2 = b_1/b_2 = c_1/c_2$ (Lines coincide)
- **No Solution:** $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (Parallel lines)
- **Speed of Boat in Still Water = x km/h, Speed of Current = y km/h**
 - Downstream Speed = $(x + y)$ km/h
 - Upstream Speed = $(x - y)$ km/h
- **Two Digit Number:** Original = $10x + y$, Reversed = $10y + x$
- **Area of Triangle formed by lines with axes:** $1/2 \times \text{base} \times \text{height}$

🔥 10. TOP 10 PRACTICE QUESTIONS FOR BOARD EXAM

- BASIC** For what value of k do equations $3x - y - 5 = 0$ and $6x - 2y - k = 0$ have no solution? (Ans: $k \neq 10$)
- BASIC** If $x = a$, $y = b$ is the solution of $x - y = 2$ and $x + y = 4$, find a and b . (Ans: $a = 3$, $b = 1$)
- MODERATE** Solve using Elimination method: $0.4x + 0.3y = 1.7$ and $0.7x - 0.2y = 0.8$. (Ans: $x = 2$, $y = 3$)
- MODERATE** Solve for x and y : $47x + 31y = 63$ and $31x + 47y = 15$. (Ans: $x = 2$, $y = -1$)

5. **ADVANCED** A fraction becomes $\frac{1}{3}$ when 1 is subtracted from numerator and becomes $\frac{1}{4}$ when 8 is added to denominator. Find the fraction. (Ans: $\frac{5}{12}$)
6. **ADVANCED** 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find time taken by 1 woman alone. (Ans: 18 days)
7. **HOTS** A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine speed of stream and boat in still water. (Ans: Boat = 8 km/h, Stream = 3 km/h)
8. **HOTS** Draw graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Calculate the area bounded by these lines and y-axis. (Ans: Area = 7.5 sq units)
9. **CBSE-STYLE** The ratio of incomes of two persons is 9:7 and ratio of expenditures is 4:3. If each saves ₹2000 per month, find monthly incomes. (Ans: ₹18,000 and ₹14,000)
10. **CBSE-STYLE** Places A and B are 100 km apart on a highway. One car starts from A and another from B at same time. If they travel in same direction, they meet in 5 hours. If towards each other, they meet in 1 hour. Find speeds. (Ans: 60 km/h and 40 km/h)

 FINAL REVISION FLOW BEFORE EXAM:

READ CONCEPT → MEMORIZE RATIO TABLE → PRACTICE ADD-SUBTRACT HACK → SOLVE WORD PROBLEMS →
SCORE 100/100!