



SUBJECT: Mathematics (Code 041)

CLASS: 10th Standard

CHAPTER: Quadratic Equations

TEACHER: Tinku Sir

EDITION: 2026-27 Board Exam

1. CHAPTER OVERVIEW & CBSE SYLLABUS BREAKDOWN

● OFFICIAL CBSE CLASS 10 SYLLABUS COVERAGE

Standard form of a quadratic equation $ax^2 + bx + c = 0$, ($a \neq 0$). Solutions of quadratic equations (only real roots) by **Factorisation** and by **using Quadratic Formula**. Relationship between discriminant and nature of roots. Situational problems based on quadratic equations related to day-to-day activities.

2. CORE CONCEPT: WHAT, WHY & HOW

① WHAT is a Quadratic Equation?

A polynomial equation of degree 2 in a single variable x is called a quadratic equation. Its standard form is:

● STANDARD FORM OF QUADRATIC EQUATION

$$ax^2 + bx + c = 0$$

Where a, b, c are real numbers and $a \neq 0$.

- a = Coefficient of x^2 (Quadratic Coefficient — MUST NOT BE ZERO!)
- b = Coefficient of x (Linear Coefficient)
- c = Constant term

② WHY do we need Quadratic Equations?

Quadratic equations model physical phenomena where quantity depends on the product/square of a variable — such as area (length × width), gravity/projectile motion (distance = $\frac{1}{2}gt^2$), revenue (price × quantity), and speed-time-distance relationships.

③ HOW to identify if an equation is Quadratic?

Simplify the given algebraic expression completely to bring all terms to LHS. If the highest power of x is exactly **2** and coefficient $a \neq 0$, it is quadratic.

● COMMON MISTAKE ALERT: HIDDEN NON-QUADRATIC / QUADRATIC FORMS

- **Mistake 1:** Thinking $(x + 1)^2 = x^2 + 3$ is quadratic. On expansion: $x^2 + 2x + 1 = x^2 + 3 \Rightarrow 2x - 2 = 0$ (Linear, not quadratic!).
- **Mistake 2:** Thinking $x + 1/x = 2$ is linear. Multiplying by x yields: $x^2 - 2x + 1 = 0$ (Quadratic!).
- **Mistake 3:** $x^2 + 3\sqrt{x} + 2 = 0$ is NOT a quadratic equation because exponent of x must be a whole number.

TINKU SIR TRICK: "DEGREE TEST & SIMPLIFY FIRST"

Never judge a quadratic equation by its cover! Always expand all brackets, remove fractions by taking LCM, collect all terms on one side, and check the highest power of x .

3. METHODS OF SOLVING QUADRATIC EQUATIONS

METHOD 1: Factorisation (Middle Term Splitting)

To solve $ax^2 + bx + c = 0$ by factorisation:

- Find two numbers p and q such that:
 - Product: $p \times q = a \times c$
 - Sum: $p + q = b$
- Split middle term bx as $px + qx$.
- Factorise by grouping into two linear factors: $(mx + n)(px + q) = 0$.
- Set each factor to zero to find roots: $x = -n/m$ or $x = -q/p$.

TINKU SIR TRICK: "SIGN RULE FOR MIDDLE TERM SPLITTING"

Sign of c (or $a \times c$)	Sign of b	Signs of factors p and q	Operation to find p, q
Positive (+c)	Positive (+b)	Both Positive (+, +)	ADD factors to get $ b $
Positive (+c)	Negative (-b)	Both Negative (-, -)	ADD factors to get $ b $
Negative (-c)	Positive (+b)	Larger (+), Smaller (-)	SUBTRACT factors to get $ b $
Negative (-c)	Negative (-b)	Larger (-), Smaller (+)	SUBTRACT factors to get $ b $

METHOD 2: Quadratic Formula (Sridharacharya's Rule)

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $D = b^2 - 4ac$ is called the **Discriminant**.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

DERIVATION OF QUADRATIC FORMULA (COMPLETING THE SQUARE)

Given $ax^2 + bx + c = 0$ ($a \neq 0$):

- Divide by a : $x^2 + (b/a)x + c/a = 0$
- Transpose constant: $x^2 + (b/a)x = -c/a$
- Add $(b/2a)^2$ to both sides: $x^2 + (b/a)x + (b/2a)^2 = (b^2/4a^2) - (c/a)$
- LHS is a perfect square: $(x + b/2a)^2 = (b^2 - 4ac) / 4a^2$

5. Take square root: $x + b/2a = \pm\sqrt{(b^2 - 4ac)} / 2a$

6. Hence: $x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$ [Proved]

ONEPATH HACK: "METHOD SELECTION MATRIX"

- If equation has small integers → **Use Middle Term Splitting** (Fastest).
- If equation has square roots (e.g. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$) → **Use Quadratic Formula** to avoid factorization confusion.
- If terms are large numbers (e.g. $100x^2 - 20x + 1 = 0$) or algebraic symbols (p, q, a, b) → **Use Quadratic Formula**.

4. DISCRIMINANT & NATURE OF ROOTS

The expression $D = b^2 - 4ac$ dictates the nature of the roots without actually solving the equation.

Value of Discriminant ()	Nature of Roots	Formula for Roots	Graphical Representation
$D > 0$ (Positive)	Two Distinct & Real Roots	$x_1 = (-b + \sqrt{D})/2a$ $x_2 = (-b - \sqrt{D})/2a$	Parabola cuts x-axis at 2 distinct points
$D = 0$ (Zero)	Two Equal & Real Roots	$x_1 = x_2 = -b / (2a)$	Parabola touches x-axis at exactly 1 point
$D < 0$ (Negative)	No Real Roots (Imaginary)	No real values	Parabola lies entirely above/below x-axis

BOARD EXAM TRAP ALERT! "REAL ROOTS" VS "DISTINCT REAL ROOTS"

- If question says: "Find k for which equation has **REAL AND EQUAL ROOTS**" → Put $D = 0$.
- If question says: "Find k for which equation has **TWO DISTINCT REAL ROOTS**" → Put $D > 0$.
- If question says: "Find k for which equation has **REAL ROOTS**" → Put $D \geq 0$ (Includes both $D > 0$ and $D = 0$!).

TINKU SIR TRICK: "THE D-CHECK FAST TRACK"

When solving for parameter k in $D = 0$ problems, if equation is $kx^2 + bx + c = 0$, remember that $a = k$ cannot be 0! Reject $k = 0$ in your final answer if it destroys the quadratic nature.

5. QUESTION IDENTIFICATION SYSTEM

"QUESTION DEKHO → TYPE PEHCHANO → METHOD CHUNO"

If Question Contains / Asks...	Identified Pattern	Action Plan / Master Formula
"Roots are equal / co-incident"	Equal Roots Condition	Set $D = b^2 - 4ac = 0$ and solve for unknown k .
"One root is reciprocal of other"	Root Relationship ($\alpha, 1/\alpha$)	Product of roots $\alpha \times (1/\alpha) = 1 \Rightarrow c/a = 1 \Rightarrow c = a$.
"One root is equal and opposite in sign"	Symmetric Roots ($\alpha, -\alpha$)	Sum of roots $\alpha + (-\alpha) = 0 \Rightarrow -b/a = 0 \Rightarrow b = 0$.
"Find quadratic equation given roots α, β "	Equation Formation	$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$
"Speed of stream / upstream-downstream"	Word Problem (Speed)	Time = Distance / Speed . Net Upstream = $v - x$, Net Downstream = $v + x$.

6. VERIFIED CBSE PYQS & MASTER SOLVED EXAMPLES

● VERIFIED CBSE PYQ 2024 [3 MARKS] — EQUAL ROOTS PARAMETER

Question: Find the value(s) of k for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has real and equal roots.

CONCEPT IDENTIFICATION:

Condition for real and equal roots: Discriminant $D = 0$.

STEP-BY-STEP SOLUTION:

Comparing with $ax^2 + bx + c = 0$:

$$a = k + 4, b = k + 1, c = 1$$

For equal roots, $D = b^2 - 4ac = 0$:

$$(k + 1)^2 - 4(k + 4)(1) = 0$$

$$k^2 + 2k + 1 - 4k - 16 = 0$$

$$k^2 - 2k - 15 = 0$$

Factorising middle term ($-5 \times 3 = -15$ and $-5 + 3 = -2$):

$$k^2 - 5k + 3k - 15 = 0 \Rightarrow k(k - 5) + 3(k - 5) = 0$$

$$(k - 5)(k + 3) = 0$$

$$k = 5 \text{ or } k = -3$$

Check: If $k = -3$, $a = -3 + 4 = 1 \neq 0$. If $k = 5$, $a = 9 \neq 0$. Both valid!

Final Answer: $k = 5, -3$

Question: A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

 STEP-BY-STEP SOLUTION:

Let speed of the stream = x km/h.

- Speed Upstream = $(18 - x)$ km/h
- Speed Downstream = $(18 + x)$ km/h
- Time Upstream $T_1 = 24 / (18 - x)$
- Time Downstream $T_2 = 24 / (18 + x)$

According to question: $T_1 - T_2 = 1$

$$24/(18 - x) - 24/(18 + x) = 1$$

Taking 24 common: $24 [(18 + x - (18 - x)) / ((18 - x)(18 + x))] = 1$

$$24 [2x / (324 - x^2)] = 1 \Rightarrow 48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

Splitting middle term ($54 \times (-6) = -324$ and $54 - 6 = 48$):

$$x^2 + 54x - 6x - 324 = 0 \Rightarrow (x + 54)(x - 6) = 0$$

Since speed cannot be negative, reject $x = -54$.

Final Answer: Speed of stream = **6 km/h**.

Question: Two water taps together can fill a tank in $9\frac{5}{8}$ hours ($75/8$ hrs). The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

 STEP-BY-STEP SOLUTION:

Let time taken by smaller tap = x hours.

Time taken by larger tap = $(x - 10)$ hours.

Part of tank filled by smaller tap in 1 hour = $1/x$

Part of tank filled by larger tap in 1 hour = $1/(x - 10)$

Combined part filled in 1 hour = $1 / (75/8) = 8/75$

$$1/x + 1/(x - 10) = 8/75$$

$$(x - 10 + x) / (x(x - 10)) = 8/75 \Rightarrow (2x - 10) / (x^2 - 10x) = 8/75$$

$$75(2x - 10) = 8(x^2 - 10x) \Rightarrow 150x - 750 = 8x^2 - 80x$$

$$8x^2 - 230x + 750 = 0 \Rightarrow 4x^2 - 115x + 375 = 0$$

Using Quadratic Formula: $a = 4, b = -115, c = 375$

$$D = (-115)^2 - 4(4)(375) = 13225 - 6000 = 7225$$

$$\sqrt{D} = \sqrt{7225} = 85$$

$$x = (115 \pm 85) / 8$$

$$x = 200/8 = 25 \text{ OR } x = 30/8 = 3.75$$

If $x = 3.75$, then larger tap time = $3.75 - 10 = -6.25$ (Impossible!). Reject $x = 3.75$.

Final Answer: Smaller tap = **25 hours**, Larger tap = **15 hours**.

7. COMPLETE CHAPTER MIND MAP

QUADRATIC EQUATIONS ($ax^2 + bx + c = 0, a \neq 0$)

1. IDENTIFICATION & STANDARD FORM

Degree must be exactly 2

Real coefficients a, b, c with $a \neq 0$

2. SOLVING METHODS

Middle Term Splitting: $p \times q = ac, p + q = b$

Quadratic Formula: $x = (-b \pm \sqrt{D}) / 2a$

3. DISCRIMINANT & NATURE OF ROOTS ($D = b^2 - 4ac$)

$D > 0 \rightarrow 2$ Real & Distinct Roots

$D = 0 \rightarrow 2$ Equal Real Roots ($x = -b/2a$)

$D < 0 \rightarrow$ No Real Roots

4. WORD PROBLEMS & APPLICATIONS

Speed-Distance: $T_1 - T_2 = \Delta t$

Tap/Work: $1/x + 1/y = 1/T$

Geometry: Area & Pythagoras Theorem

8. ONEPATH ONE-PAGE REVISION SHEET

EVERYTHING YOU NEED FOR LAST MINUTE REVISION

Standard Form	$ax^2 + bx + c = 0, (a \neq 0)$
Discriminant	$D = b^2 - 4ac$
Quadratic Formula	$x = (-b \pm \sqrt{D}) / (2a)$
Equal Roots Condition	$b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$
Sum & Product of Roots	$\alpha + \beta = -b/a, \quad \alpha \times \beta = c/a$
Form Quadratic Eq.	$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$
Upstream/Downstream	Upstream = $v - x$, Downstream = $v + x$

9. STUDENTS OFTEN MAKE THESE MISTAKES

MISTAKE 1: FORGETTING TO PUT $\pm$ WHEN TAKING SQUARE ROOT

Wrong: $(x - 3)^2 = 16 \Rightarrow x - 3 = 4 \Rightarrow x = 7$ (Loses one root!)

Correct: $(x - 3)^2 = 16 \Rightarrow x - 3 = \pm 4 \Rightarrow x = 7$ or $x = -1$.

MISTAKE 2: SIGN ERROR IN QUADRATIC FORMULA WHEN B IS NEGATIVE

If $b = -7$, $-b = -(-7) = +7$. Many students write -7 !

MISTAKE 3: NOT REJECTING INADMISSIBLE VALUES IN WORD PROBLEMS

Speed, time, age, and lengths **CANNOT BE NEGATIVE**. Always explicitly state: "Rejecting negative value as speed/length cannot be negative."

10. TOP 10 MUST-PRACTICE QUESTIONS

- BASIC** Solve for x : $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$.
- BASIC** Find k if equation $2x^2 + kx + 3 = 0$ has equal roots.
- MODERATE** Solve by factorisation: $4x^2 - 4ax + (a^2 - b^2) = 0$.
- MODERATE** Solve for x : $1/(x+4) - 1/(x-7) = 11/30$ ($x \neq -4, 7$).

5. **ADVANCED** If roots of equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $a/b = c/d$.
6. **ADVANCED** A train travels 360 km at a uniform speed. If speed had been 5 km/h more, it would have taken 1 hour less. Find speed.
7. **HOTS** Solve for x: $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$.
8. **BOARD** A peacock is sitting on top of a pillar 9m high. From a point 27m away from the bottom, a snake is coming. Snake speed = Peacock speed. At what distance from hole are they met?
9. **BOARD** Sum of areas of two squares is 468 m². If difference of perimeters is 24 m, find sides.
10. **HOTS** If -5 is a root of $2x^2 + px - 15 = 0$ and $p(x^2 + x) + k = 0$ has equal roots, find k.

11. FINAL CHAPTER REVISION CHECKLIST

- ☐ Standard Form & Degree test clear
- ☐ Middle term splitting sign rules memorized
- ☐ Quadratic formula & Discriminant derivation understood
- ☐ Nature of roots conditions ($D > 0$, $D = 0$, $D < 0$) practiced
- ☐ Speed-Distance-Time & Upstream/Downstream setup practiced
- ☐ Tap & Tank fraction setup practiced
- ☐ Tinku Sir Tricks & OnePath Hacks reviewed
- ☐ Common mistakes noted

 **POWERED BY TINKU SIR TRICKS × ONEPATH ACADEMY • ALL RIGHTS RESERVED**